# Special Session
# "Set-Valued Numerics and Reliable Computing"

*Sozopol, Bulgaria, June 4–8, 2003*

As part of the 4-th International Conference on "Large-Scale Scientific Computations" (`http://parallel.bas.bg/~scicom03/`), a special session on Set-Valued Numerics and Reliable Computing was organized by Neli Dimitrova, Mikhail Krastanov, and Vladimir Veliov.

One of the main problems of interval computations is computing the range of a known function over known intervals. For polynomials, S. Miyajima and M. Kashiwagi (Waseda U., Shinjuku-ku, Japan) described and compared several new methods that combine, in different ways, known ideas (Horner's method, bisection, Taylor series, etc.). For special functions, even reliable computation of a single value at a single point is often a challenging computational problem; for this problem, A. Cuyt, B. Verdonk, H. Waadeland, and J. Vervloet (U. of Antwerp, Belgium) described a new efficient and reliable algorithm based on continuous fractions. Their main idea is that instead of using the first few terms in the continuous function expansion, we can start with an appropriately pre-computed values and thus, speed up the computations.

One of the main reasons why we need data processing in the first place is that we want to use the results of data processing in decision making. For example, in many application areas (e.g., in fault detection), we must detect outliers. V. Kreinovich, L. Longpré (U. Texas, El Paso), S. Ferson, and L. Ginzburg (Applied Biomathematics, USA) describe algorithms for detecting outliers under interval uncertainty.

In many practical problems, intervals are not enough. One reason is that, e.g., when the interval of possible values of $b$ contains 0, then the set of possible values of the ratio $a/b$ is no longer an interval. B. Verdonk, J. Vervloet, and A. Cuyt (U. of Antwerp, Belgium) show how we can extend the set of all intervals with such non-interval sets and how to efficiently implement the resulting "vset" arithmetic.

Another reason why we need to go beyond intervals is that traditional interval computations do not always take into consideration the dependence between the intermediate results, and thus, the resulting interval enclosures have excess width. One way of decreasing this excess width is to explicitly keep track of how the intermediate results depend on the input parameters; this method was efficiently used by E. Popova, M. Datcheva, R. Iankov (Bulgarian Academy of Sciences) and

T. Schanz (Bauhaus U., Weimar, Germany) to produce sharp bounds for strains and stresses in mechanical systems.

Another possibility is to describe the uncertainty on each intermediate step not simply by keeping an interval of possible values for each variable, but also by keeping an information about a multi-D set of all possible vectors consisting of these variables. Among several useful ways of describing such sets are zonotopes (Minkowski sums of intervals). S. Markov and D. Claudio showed how we can approximate an arbitrary zonotope by a finite-dimensional family of zonotopes, a family whose elements are easy to process because they have nice properties similar to the properties of a vector space. C. de la Mora, P. Wojciechowski, O. Kosheleva, V. Kreinovich, and S. A. Starks (U. Texas-El Paso) showed how similar multi-D sets can be used to describe uncertainty in expert preferences, and how the resulting techniques can be used to characterize system response to damage.

In many applications areas, there is an additional complication caused by the fact that the systems that we want to control and analyze are dynamical systems, often with non-linear behavior. In their talk, N. S. Dimitrova and M. I. Krastanov (Bulgarian Academy of Sciences) handled one such challenging problem: they described stabilizing feedback of a nonlinear biological wastewater treatment plant.

One of the methods for analyzing strongly non-linear dynamics is a method of Volterra series, a kind of generalization of Taylor series from functions to functionals in which each functional is represented as a series consisting of a linear, quadratic, cubic, etc. parts. N. Kirov and M. Krastanov (Bulgarian Academy of Sciences) showed that this method can lead not only to heuristic estimates, but to guaranteed bounds on the distance between the approximate and actual trajectories of the dynamical system.

V. Veliov (Vienna U. of Technology, Austria, and Bulgarian Academy of Sciences) described how to solve dynamical systems in which the dynamics of each element depends not only on its own state and control but also on some global characteristics of the entire system (economics is a good source of examples of such systems).

Two talks described systems in which (like in the analysis of infectious diseases) the dynamics depends on the age of the individual. General methods for solving such systems were presented by C. Almeder (Vienna U. of Technology, Austria); application of fish growth in captivity were given by J. Hernandez and E. Gasca-Leyva (U. of Las Palmas, Spain).

In control, there is often an additional complication—that even for smooth dynamical systems, the optimal control is often discontinuous; there is even an official term: "bang-bang control." In his talk, I. Chryssoverghi (National Technical U. of Athens, Greece) described how gradient-based techniques can be extended to such discontinuous (hence non-differentiable) situations.

Finally, in some practical cases, the uncertainty with which we know dynamics is so high that instead of differential equations that describe the next state in terms of the previous ones, we have differential inclusions that only describe the set

of possible next states. N. Pulova and V. Pulov (Technical U. Varna, Bulgaria) showed how differential-based Runge-Kutta-type techniques of solving systems of differential equations can be extended to cover differential inclusions as well.

Most of the presented papers have been accepted for publication in a special volume of Springer Lecture Notes in Computer Science.

The conference was held in Sozopol, a beautiful ancient small town on the Black Sea coast. The town was founded in the 7th century B.C. on a small peninsula near a convenient bay, it was the first Hellenic colony on the Black Sea shore.

The conference hotel was located right on the beach, walking distance from the old city, so, in addition to interesting talks, we all enjoyed swimming in the calm majestic sea, visiting the old town with picturesque narrow streets and old houses that still preserve their old Bulgarian architecture.

On June 6, we had a tour of Old Nesebur, a nearby UNESCO-protected ancient town with numerous temples, churches, and statues dating from 6th century B.C. to 8th century A.D.

On June 7, we had a lot of fun during the banquet. We watched, in awe, an exciting spectacle of people walking barefoot over burning coals. After that, Bulgarian folk dancers and musicians not only entertained us with their fiery dances, but they also made everybody dance. It was unforgettable!

*Svetoslav Markov and Vladik Kreinovich*